

## INVESTIGATING WHAT MAKES BEGINNING TEACHERS' ENACTMENT OF NUMBER TALKS MORE OR LESS AMBITIOUS

Jillian M. Cavanna      Byungeun Pak      Brent Jackson  
 University of Hartford    Dixie State University    Michigan State University  
 cavanna@hartford.edu    byungeun.pak@dixie.edu    b.jackson@msu.edu

*Number talks, a popular mathematics teaching routine in the United States, may offer supports for beginning teachers (BTs) to engage in ambitious instruction. BTs' enactments of number talks, however, are varied, and there are few empirical studies that explore how BTs' enactment of number talks could be more (or less) ambitious. This paper draws on classroom observation data from a large investigation of BTs' enactment of ambitious instruction in elementary mathematics across five teacher preparation programs. We analyzed 19 transcripts of number talks enacted by seven BTs to investigate what makes number talks more or less ambitious. Findings illustrate three patterns among number talks that were categorized as approaching ambitious using the M-Scan (Berry et al., 2013) instrument. Discussions and implications are offered in relation to mathematics teacher education and research.*

**Keywords:** Elementary School Education, Instructional Activities and Practices, Teacher Educators

Beginning teachers (BTs) are expected to engage in ambitious mathematics instruction (Kazemi, Franke, and Lampert, 2009; Lampert et al., 2013) that is also equitable for all students (e.g., Jackson & Cobb, 2010). This kind of teaching is inherently complex. For example, it requires teachers to be responsive to their students relative to both mathematics content and pedagogical methods, while also attending to unique social management demands required for productive mathematical classroom discourse (Lampert, Beasley, Ghuousseini, Kazemi, & Franke, 2010).

Number talks, instructional routines in which students use mental mathematics to solve computational problems, offer one way for BTs to engage in ambitious and equitable instruction. Number talks create a participation structure that may support “students to take back the authority of their own reasoning” (Humphreys & Parker, 2015, p. 1). The *routinized* nature of number talks can offer strong support for BTs to engage in aspects of ambitious instruction as novices. Despite the growing popularity of number talks in elementary classrooms there is a need for more empirical evidence about their enactment (Matney, Lustgarten, & Nicholson, 2020). In our recent work (Authors, 2021a, 2021b, and 2021c), we observed variation in BTs' enacted number talks in both structure and equitable opportunities offered to students. We continue this work to explore the features of BTs' enactment of number talks to answer the research question: What makes BTs' number talks more or less ambitious?

### Theoretical Framework

To frame BTs' enactment of number talks, we build from the perspective that *ambitious instruction* in mathematics consists of teaching practices that foster students' deep, conceptual understanding of content (Lampert et al., 2013). Rigor, a hallmark of ambitious instruction, is evident in teachers' selection of tasks, supports for students, and how they respond to what students think and do (Kazemi et al., 2009). Further, when teaching ambitiously, teachers focus students on intellectual processes that support them to tackle demanding tasks, pushing students

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Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

to justify their approaches, and pressing them to elaborate on their explanations and to clarify their thinking. Since the number talk routine provides space for students to engage in these kinds of mathematical practices, we see enacting number talks as a support for BTs to engage in ambitious teaching practices.

Additionally, we draw on Cazden's (2001) perspective on classroom interaction, which is shaped by two dimensions (*sequential* and *selectional*) of teaching. The *sequential dimension* describes relatively stable, cultural structure and routines. In contrast, the *selectional dimension* describes how individuals navigate such structures and routines. The *selectional dimension* highlights teachers' agency within a set of (taken for granted-or-not) constraints. Similar to Ehrenfeld and Horn's (2020) utilization of these two dimensions to describe teachers' group work monitoring routines, we view number talk routines as shaped by both *sequential* and *selectional* dimensions of teaching. For example, Parker and Humphrey's (2018) description of the "Revised Number Talk Routine" (p. 40) highlights the *sequential* dimension inherent in number talks. They found a strong correlation between teachers' decision-making within the number talk routine and the quality of teachers' number talks. Similarly, we found that BTs followed a predictable structure of enacting consistent phases in their number talks, *introducing*, *collecting*, *idea sharing*, and *closing* (See Figure 1; Cavanna et al., 2021; Pak et al., 2021a; and Pak et al., 2021b). Additionally, we identified important variations with the *idea sharing* phase on this structure (Pak et al 2021a), which highlights the *selectional* dimension of NT routines. Specifically, we found that number talks that included particular types of segments during the idea sharing phase created more opportunities for multiple students to engage with mathematical ideas than were available in number talks with only simple strategy segments shared by a single student. In this study we further explore the *selectional* dimension to understand the implications of BTs choices on the ambitiousness of their lessons.

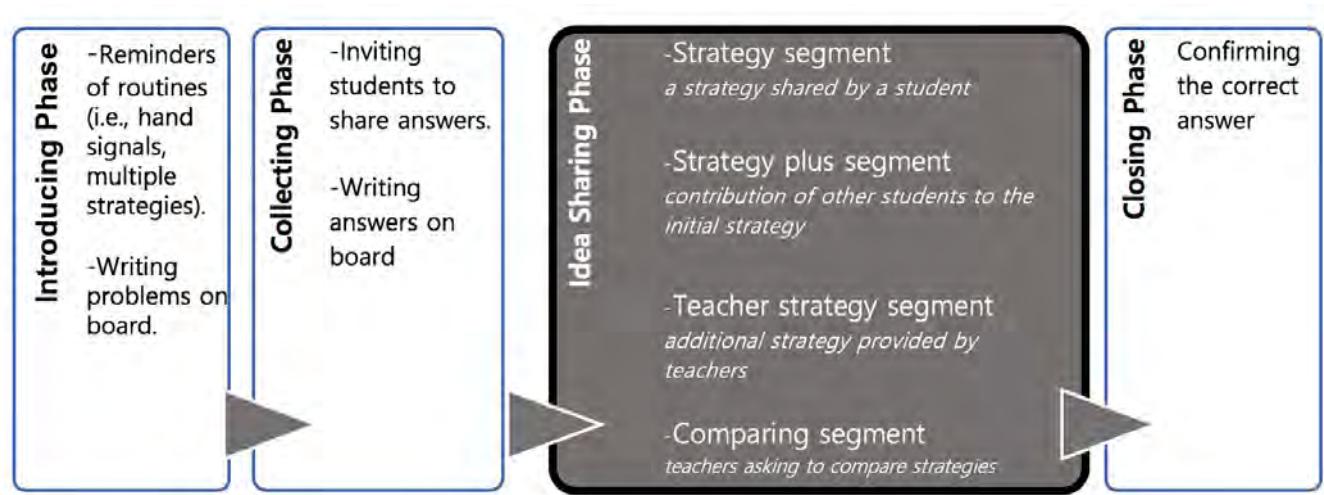


Figure 1: Number Talks Routine Phases

## Methods

### Context and Data Sources

Data for this study was gathered as part of a large research project that investigated BTs' enactment of instructional practices in elementary mathematics and English/language arts (ELA). Research project members video recorded BTs (n=69) three times per year as they taught

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mathematics and ELA. For this study, we draw on video data from a subset of 16 purposefully selected case-study participants. We reviewed 144 videos from these case-study participants (three mathematics lessons per year) over the course of three years. Of these, we identified 19 videos in which seven case-study teachers enacted Number Talks. The Number Talks occurred at the beginning of each lesson, with an average duration of ten minutes per number talk. These 19 number talk transcripts comprise the data for this investigation.

### Data Analyses

As part of the larger project, all video recorded lessons were scored using the Mathematics Scan (M-Scan) instrument (Berry et al., 2013). This instrument offered an assessment of the ambitiousness of the mathematics instruction (Berry et al., 2010). Of the nine elements assessed by M-Scan, we focused on six categories of ambitious teaching: cognitive demand, problem solving, use of representations, mathematical discourse community, explanation and justification, and mathematical accuracy. These six dimensions capture key elements of ambitious mathematical instruction (Kazemi et al., 2009; Lampert et al., 2013). Scores on the M-Scan instrument range from 1 to 7, with scores of 6 or 7 considered to be demonstrating the most ambitious mathematical instruction. We considered mean scores across the six focal dimensions that were greater than 5 to be approaching ambitious instruction. Although the M-Scan scores were evaluated based on the full lesson and the number talks typically comprised less than that, we considered the instruction of the number talks portion as having a significant effect on the overall lesson score.

Our analyses of the number talk transcripts involved iterative stages of qualitative coding by the three authors of this study. First, to understand the *sequential* dimension of the number talks, we divided transcripts into phases (see Figure 1; Pak et al., 2021a). Next, we explored the *selectional* dimension of the number talks. We divided the *Idea Sharing* phase into segments, which were separated by the span in which teachers allow students to talk about a particular mathematical idea. We closely examined each segment in this phase because our prior works suggested that a systematic analysis of this *Idea Sharing* phase might show what made BTs' number talks more (or less) ambitious. The authors iteratively coded 19 transcripts, comparing individual codes until we reconciled all coding across the team. Table 1 outlines the segment types identified as a result of this process. We recognize the similarity of the *Strategy Plus* subcodes to well-known mathematical talk moves (e.g., Chapin, O'Connor, Anderson, 2009). In our presentation we expand on these points of convergence and divergence from existing literature.

**Table 1. Codes to represent various teacher moves within Idea Sharing phase**

Segment Types	Descriptions and subcodes	
Strategy	Asking students to volunteer talking about their strategies or calling on a specific student to explain a strategy to solve a problem.	
Strategy plus	Inviting, or calling on, students to share their ideas related to how they make sense of the initial strategy.	(1) Inviting Directly inviting students to question the strategy sharer.

	(2) Interpreting	Asking another student to offer his/her own reasoning regarding how the initial strategy works.
	(3) (Dis)agreeing	(Dis)agreeing with follow-up (Not related to an error)
	(4) Detecting	Detecting error and may challenge the strategy sharer
	(5) Guiding	Generating the reasoning to draw attention to something specific in the strategy (e.g., funneling).
	(6) Repeating	Asking students to revoice what another student says
Teacher strategy	Initiating and providing additional strategies to solve the problem.	
	(1) Feeding a strategy	Making moves to push students to consider thinking in a certain way, but don't set up the problem in a way to use a certain strategy. (e.g., What if...Could I...?)
	(2) Do this strategy	Setting up the problem to be carried out a particular way.
	(3) Call and response	Walking a student (or the class in choral response) through a logical progression.
	(4) Guess my strategy	After asking students to guess, teachers explain what he/she did and ask students to unpack the reasoning.
Comparing Strategy	Asking students to compare similarities and differences between strategies.	

Lastly, in order to answer the question of what makes number talks more (or less) ambitious, we examined the segment types present across the set of 19 lessons and compared those to the M-Scan scores on those lessons. Specifically, we developed matrices that recorded the types and frequencies of segments within each number talk. We then looked for patterns across the lessons related to the M-Scan scores and patterns within those number talks.

## Results

Our analyses of the number talk transcripts and M-Scan scores revealed six of 19 number talks as approaching ambitious mathematics instruction. Figure 2 lists these lessons and the mean scores of the M-Scan dimensions we utilized. Across this set of lessons, we identified three salient patterns that seem to be associated with approaching ambitious number talks. In the following sections we draw on excerpts from these six more ambitious lessons and compare to the other 16 less ambitious number talks to help us understand what makes some number talks more or less ambitious.

The first pattern we observed relates to the prevalence of *Strategy Plus* segment types across the more ambitious number talks. As shown in Figure 2, *Strategy Plus* segments appeared frequently across the *Idea Sharing* phase of the approaching ambitious number talks. Within the strategy plus segments, we also identified further variation across different teacher moves, including *Inviting*, *(Dis)agreeing*, *Interpreting*, *Guiding*, and *Detecting* (See Table 1). For

example, Teacher B, in lesson Y1 M2, used both the *inviting* and *interpreting* moves in two adjacent *Strategy Plus* segments. Teacher B began the segment by inviting a student Jason to share an initial strategy. Jason responded to the invitation and explained his strategy for solving the addition problem  $470 + 450$ . Next, the teacher *invited* the class to question Jason's work, saying "Does anyone have a question about that?". This question prompted two students to pose questions about the mathematics in Jason's strategy. Following this exchange, the teacher used an *interpreting* move to support one of the questioning students, Landon, to go further in unpacking Jason's initial strategy. The short excerpt below illustrates this *interpreting* move and the subsequent exchange.

Landon: 450 plus 400 which is 850

Teacher B: [writing on board] 850, okay.

Landen: So, we're kind of back where we started up here a little bit? Right? Wait, why would we plus 30?

Teacher B: We would plus 50, right? Why? Landen?

Landen: Because you can benchmark.

This exchange from Teacher B is characteristic of the kinds of inviting and interpreting moves we observed in five of the six ambitious number talks. As illustrated in this exchange, BTs tended to use multiple *Strategy Plus* talk moves within these more ambitious lessons to move support multiple students to engage with the mathematical strategies being shared.

Lesson (Year, Obs)	M-Scan Means <sup>2</sup>	NT Idea Phase Segments						
Teacher A (Y2 M1)	5.0	Strategy	Strategy Plus [Interpreting]	Strategy Plus [Detecting]	Strategy			
Teacher B (Y1 M2)	5.3	Strategy	Strategy	Strategy	Strategy	Strategy	Strategy	Teacher Strategy [Do This Strategy]
		Strategy Plus [Inviting]	Strategy Plus [Interpreting]	Teacher Strategy [Guess My Strategy]	Strategy	Strategy		
Teacher B (Y1 M3)	5.2	Strategy	Strategy	Strategy Plus [Interpreting]	Strategy	Strategy	Teacher Strategy [Do This Strategy]	Strategy Plus [Inviting]
		Strategy						
Teacher C (Y2 M1)	5.2	Strategy Plus* [Guiding]	Strategy Plus* [(Dis)agreeing]	Strategy	Strategy Plus* [Interpreting]	Strategy Plus* [Detecting]		
Teacher C (Y2 M2)	5.3	Strategy	Strategy Plus* [Detecting]					
Teacher C (Y3 M2)	5.3	Teacher Strategy* [Do This Strategy]	Strategy Plus* [Inviting]	Strategy Plus* [Inviting]	Strategy Plus [Inviting]	Comparing		

Note. \* indicates the segment contained a mathematical error

**Figure 2. Idea Sharing Phase Segments of Ambitious Number Talks**

The second pattern we observed across this set of number talks was related to the ways BTs inserted their own strategies into number talk discussions. We call this kind of move *Teacher Strategy* segments. While not all examples of teacher strategy segments were associated with approaching ambitious instruction, we observed two subtypes of teacher strategy segments within the ambitious number talks: (1) *Do This Strategy* and (2) *Guess My Strategy*. The excerpt

below illustrates how Teacher C set up a problem to be carried out in a specific way through an example of the *Do This Strategy* teacher move.

Teacher C: First of all, before we start sharing out strategies and agreeing and disagreeing with people. I'm wondering how we would put this into a bar diagram [pointing to diagram on board] because a bar diagram is a super important tool. Why is a bar diagram such an important tool? John, why is it an important tool?

John: Because you can use it to know parts and wholes.

Teacher C: Yeah, you can use it to understand your parts and your whole. And if you're going to add or you are going to —

Class/Teacher C: subtract.

Teacher C: And if this isn't a word problem, we know right away we are going to do what to solve this?

Class: subtract.

Following this excerpt, students worked the number talk routine, with many students utilizing bar models in their solutions. This excerpt shows how Teacher C intentionally encouraged students to use a bar model to solve a problem. Prior to solving the problem, however, Teacher C supported students to understand why a bar model might be helpful in solving this particular problem. We observed the *Do This Strategy* move in three number talks with approaching ambitious M-Scan scores (i.e., Teacher B (Y1 M2), Teacher B (Y1 M3), Teacher C (Y3 M2)).

In contrast, we observed a different teacher strategy, which we called, *Feeding Strategy*, only in number talks that were not ambitious according to their M-Scan scores. Due to space constraints, we did not present the less ambitious number talks from our dataset in Figure 2. We will expand upon the contrasting features of the more and less ambitious Number Talks in our presentation. To illustrate the differences between more and less ambitious teacher strategies, we offer an excerpt from Teacher D (Y2 M1), which includes a *Feeding Strategy*.

Teacher D: Did anybody use the distributive property, but you split it up in a different way?

Or can you think of another way that would make sense to split it up? Ally?

Ally: I split up the 12 into 6 and 6. And did 6 times 3 and 6 times 3.

Teacher D: Yeah - So, if you split the 12 into 6 plus 6, then you have 6 times 3 plus 6 times 3. If you love your six facts. Maybe you don't know your 6 times 6. What's 6 times 3?

Class: 18.

Teacher D: 18 plus 18 is?

In this excerpt, Teacher D pressed students to think about how to solve the problem in a certain way (e.g., using the distributive property). This teacher, however, did not support students to reason about why this specific solution strategy might be helpful in this problem. We found it somewhat surprising that these *Feeding Strategy* appeared only in number talks that were not ambitious, while the *Do This Strategy*, another type of teacher strategy, appeared in ambitious number talks. We wonder if perhaps the nuances between the BTs setting students up with a possible solution strategy in advance, as was the case for the *Do This Strategy* move, offered more space for teachers to build from students' thinking than when BTs funneled students towards a particular strategy later in the number talk, as in the *Feeding Strategy* move. Further investigation is warranted.

The third and final pattern we observed relates to the role of mathematical errors in number talks. As noted in Figure 2, segments with an asterisk symbol (\*) contain an error; there are three

ambitious number talks in which students made mathematical errors. All three were facilitated by Teacher C. Across these number talks, we observed errors in both *Strategy Plus* segments and *Teacher Strategy* segments. Teacher C used a range of moves to respond to errors made by students, *Inviting*, *Guiding*, *(Dis)agreeing*, and *Interpreting*. Importantly, Teacher C demonstrated ambitious mathematics instruction in the ways she supported students to make sense of the mathematics related to the errors. For example, in Teacher C's (Y2 M1) lesson, many students shared incorrect answers to the subtraction problem 74–36. The excerpt below shows how Teacher C (Y2 M1) used a *Guiding* move to respond to these errors.

Teacher C: So, it's already up here, alright, excellent, we have four different ideas, we have four different ways. What were we just doing when we were just thinking about that?

James: Most of us were using that.

Teacher C: Maybe we were using our algorithm. Who could walk me through what you did?

[Many hands in the air] Who can walk me through it? Talk it up. Alex, what did you do?

Alex: First, I did regrouping.

Teacher C: Okay. How did you do that?

In this lesson, Teacher C received three different incorrect answers and one correct answer. The teacher guided the students to use “our algorithm” to solve the problem. The teacher asked Alex, one of the students who shared an incorrect answer, to walk the class through what he did to get the answer (“ideas”). This excerpt is characteristic of Teacher C’s student-centered approach to handling students’ incorrect answers.

### Discussions and Conclusion

Our analyses revealed three patterns related to BTs’ number talks being more ambitious, all of which related to teacher moves occurring within the *Idea Sharing* phase of the number talks routine. Specifically, we observed patterns related to the nature of *Strategy Plus* segments, *Teacher Strategy* segments, and the ways BTs responded to student errors. These findings offer potential insights for the field of mathematics teacher education as we seek to support BTs to engage in ambitious mathematics instruction.

First, these findings suggest that number talks offer a transportable container for BTs to engage in ambitious instructional practices early in their teaching career. At the same time, we found that only six of 19 number talk lessons could be considered examples of ambitious mathematics instruction. Therefore, we posit that the number talk routine itself—the *sequential* dimension—does not necessarily result in ambitious instruction. Instead, systemic analysis of BTs’ number talks in terms of the *selectional* dimension (Cazden, 2001) offers insights into the role of teacher choice within the number talk container. It is within the *selectional* dimension that BTs exercised their instructional agency that created opportunities for the number talks to be characterized as more or less ambitious. This suggests that mathematics teacher educators must not only work with BTs to understand and use the overall number talk structure, or container, but also to consider the way certain segments are used within the *Idea Sharing* phase. This relates to our next point of discussion.

Second, these findings suggest that if we want BTs to engage in more ambitious number talks, then they need to move beyond strings of *strategy* segments, in which students simply share out one strategy after another. Instead, we offer that BTs need to integrate intentional talk moves (e.g., Chapin, O’Connor, Anderson, 2009; Kazemi & Hintz, 2014) to bring in multiple students into the conversation and shift towards *Strategy Plus* segments, as well as offer *Teacher*

*Strategy* segments. Such findings offer implications for mathematics teacher education. Specifically, mathematics teacher educators need to provide opportunities for BTs to learn to infuse *Strategy Plus* segments into their number talk lessons. Further research into the role that particular arrangements of talk moves play in the ambitiousness of number talks is warranted.

Third, our findings do not highlight *Comparing* segments as clearly tied to the ambitiousness of number talks. Building on prior literature, we anticipated that *Comparing* segments, which offer opportunities for students to build off of another's reasoning (e.g., Herbel-Eisenmann, Steele, Cirillo, 2013; Waggoner, 2015) would be associated with ambitious number talks. Interestingly, we observed only one instance of a *comparing* segment, in Teacher C (Y3 M2). Since creating opportunities for students to build on one another's reasoning is a potentially difficult move, perhaps this is not so surprising given our dataset was lessons enacted by teachers within their first three years of teaching. At the same time, these findings may suggest that BTs could benefit from more opportunities to learn about moves to support students to compare mathematical reasoning. Such support could, in turn, contribute to more ambitious number talks.

Lastly, number talks are increasingly popular; and due to their portability and ease of use, they will likely continue to be a staple of mathematics classrooms. The elements highlighted in this study offer possibilities for enhancing the power of these flexible routines. Specifically, number talks that are more ambitious offer greater opportunities for engaging students in critical disciplinary practices of mathematics, such as the Standards for Mathematical Practice (NGA & CCSSO, 2010). At the same time, in number talks that are more ambitious, we see more space for equitable mathematical opportunities, as well. There is a need for further research that examines BTs enactment of number talks over time in order to understand how the ways students' mathematical identities develop and the ways number talks affect the nature of power structures within the mathematics classroom. Number talks that are both ambitious and equitable offer a route for teachers to provide meaningful opportunities for *all* students to engage in critical mathematical practices.

### Acknowledgments

This material is based upon work supported by grants from the National Science Foundation (NSF; grant no. DGE1535024) and the Spencer Foundation (grant no. 201600103). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the funders.

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Olanoff, D., Johnson, K., & Spitzer, S. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.